

# Radiating black hole solutions in arbitrary dimensions

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We prove a theorem that characterizes a large family of non-static solutions to Einstein equations in  $N$ -dimensional space-time, representing, in general, spherically symmetric Type II fluid. It is shown that the best known Vaidya-based (radiating) black hole solutions to Einstein equations, in both four dimensions (4D) and higher dimensions (HD), are particular cases from this family. The spherically symmetric static black hole solutions for Type I fluid can also be retrieved. A brief discussion on the energy conditions, singularities and horizons is provided.

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## I. INTRODUCTION

In recent time, it was demonstrated that the string theory requires higher dimensions for its consistency. Models with the space-time with large extra dimensions were recently proposed in order to solve the hierarchy problem, that is to explain why the gravitational coupling constant is much smaller than the coupling constants of other physical interactions. These new concepts of higher dimensional physics have a number of interesting applications in modern cosmology and theory of gravity. This has triggered, during the past decade, a significant increase in interest in black holes in higher dimensions (HD) (see, for example, the review articles of Horowitz [1] and Peet [2]). There is now an extensive literature of solutions in string theory with horizons, which represent black holes, and related objects in arbitrary dimensions. The physical properties of these solutions have been widely studied. Interest in black holes in HD has been further intensified in recent years, due, for example, to the role they have played in the conjectured correspondence between string theory (or supergravity) on asymptotically locally anti-de Sitter backgrounds and the large- $N$  limit of certain conformal field theories defined on the boundary-at-infinity of these backgrounds [3, 4, 5].

Static and spherically symmetric space-times are one of the simplest kinds of space-times that one can imagine in general relativity. Yet, even in this simple situation, solving the Einstein field equations may be far from trivial. In fact, it turns out that such a problem is intractable due to complexity of the Einstein field equations. Hence, there are very few inhomogeneous and nonstatic solutions known, one of them is the Vaidya solution. The Vaidya solution [6] is a solution of Einstein's equations with spherical symmetry for a null fluid (radiation) source (a Type II fluid) described by energy momentum tensor  $T_{ab} = \psi l_a l_b$ ,  $l_a$  being a null vector field. The Vaidya's radiating star metric is today commonly used for two purposes: (i) As a testing ground for various formulations of the Cosmic Censorship Conjecture (CCC). (Actually CCC is a famous conjecture, first formulated by Penrose [7]. The conjecture, in its weak version, essentially state that any naked singularity which is created by evolution of regular initial data will be shielded from the external view by an event horizon. According to the strong version of the CCC, naked singularities are never produced, which in the precise mathematical terms demands that space-time should be globally hyperbolic.) (ii) As an exterior solution for models of objects consisting of heat-conducting matter. Recently, it has also proved to be useful in the study of Hawking radiation, the process of black-hole evaporation [8], and in the stochastic gravity program [9]. It has also advantage of allowing a study of the dynamical evolution of horizon associated with a radiating black hole.

Also, several solutions in which the source is a mixture of a perfect fluid and null radiation have been obtained in later years [10]. This includes the Bonnor-Vaidya solution [11] for the charge case, the Husain solution [12] with an equation of state  $P = k\rho$ . Glass and Krisch [13] further generalized the Vaidya solution to include a string fluid, while

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charged strange quark fluid (SQM) together with the Vaidya null radiation has been obtained by Harko and Cheng [14] (see also [15]). Wang and Wu [16] further extrapolated the Vaidya solution to more general case, which include a large family of known solutions.

Motivated by this and by a recent work Salgado [17], we [18] have proved a theorem characterizing a three parameter family of solutions, representing, in general, spherically symmetric Type II fluid that includes most of the known solutions to Einstein field equations. This is done by imposing certain conditions on the energy momentum tensor (EMT) (see also [19, 20, 21, 22]).

In this paper, we consider an extension of our work [18], so that a large family of exact spherically symmetric Type II fluid solutions, in arbitrary dimensions, are possible, including it's generalization to asymptotically de Sitter/anti-de Sitter.

## II. THE RADIATING BLACK-HOLE SOLUTIONS

**Theorem - I:** Let  $(M, g_{ab})$  be an  $N$ -dimensional space-time [ $\text{sign}(g_{ab}) = +(N-2)$ ] such that (i) It is non-static and spherically symmetric, (ii) it satisfies Einstein field equations, (iii) in the Eddington-Bondi coordinates where  $ds^2 = -A(v, r)^2 f(v, r) dv^2 + 2\epsilon A(v, r) dv dr + r^2(d\Omega_{N-2})^2$ , where,  $(d\Omega_{N-2})^2 = d\theta_1^2 + \sin^2(\theta_1)d\theta_2^2 + \sin^2(\theta_1)\sin^2(\theta_2)d\theta_3^2 + \dots + \left[\left(\prod_{j=1}^{N-2} \sin^2(\theta_j)\right) d\theta_{N-1}^2\right]$ , the energy-momentum tensor  $T^{ab}$  satisfies the conditions  $T_r^v = 0$  and  $T_{\theta_1}^{\theta_1} = kT_r^r$ , ( $k = \text{const.} \in \mathcal{R}$ ) (iv) it possesses a regular Killing horizon or a regular origin. Then the metric of the space-time is given by

$$ds^2 = - \left[ 1 - \frac{2m(v, r)}{(N-3)r^{N-3}} \right] dv^2 + 2\epsilon dv dr + r^2(d\Omega_{N-2})^2, \quad (\epsilon = \pm 1) \quad (1)$$

where

$$m(v, r) = \begin{cases} M(v) & \text{if } C(v) = 0, \\ M(v) - 8\pi C(v) \left( \frac{N-3}{N-2} \right) \frac{1}{(N-2)k+1} r^{(N-2)k+1} & \text{if } C(v) \neq 0 \text{ and } k \neq -1/(N-2), \\ M(v) - 8\pi C(v) \left( \frac{N-3}{N-2} \right) \ln r & \text{if } C(v) \neq 0 \text{ and } k = -1/(N-2). \end{cases} \quad (2)$$

$$T_b^a = \frac{C(v)}{r^{(N-2)(1-k)}} \text{diag}[1, 1, k, \dots, k]. \quad (3)$$

and

$$T_v^r = \begin{cases} \frac{1}{8\pi r^{N-2}} \left( \frac{N-2}{N-3} \right) \frac{\partial M}{\partial v} - \frac{1}{(N-2)k+1} \frac{\partial C}{\partial v} r^{(N-2)(k-1)+1} & \text{if } k \neq -1/(N-2), \\ \frac{1}{8\pi r^{N-2}} \left( \frac{N-2}{N-3} \right) \frac{\partial M}{\partial v} - \frac{1}{r^{N-2}} \frac{\partial C}{\partial v} \ln r & \text{if } k = -1/(N-2). \end{cases} \quad (4)$$

Here,  $M(v)$  and  $C(v)$  are the arbitrary functions whose values depend on the boundary conditions and the fundamental constants of the underlying matter.

**Proof:** Expressed in terms of Eddington coordinate, the metric of general spherically symmetric space-time in  $N$ -dimensional space-times [23, 24, 25, 26] is,

$$ds^2 = -A(v, r)^2 f(v, r) dv^2 + 2\epsilon A(v, r) dv dr + r^2(d\Omega_{N-2})^2. \quad (5)$$

Here  $A(v, r)$  is an arbitrary function. It is useful to introduce a local mass function  $m(v, r)$  defined by  $f(v, r) = 1 - 2m(v, r)/(N-3)r^{(N-3)}$ . For  $m(v, r) = m(v)$  and  $A = 1$ , the metric reduces to the  $N$ -dimensional Vaidya metric [23].

In the static limit, this metric can be obtained from the metric in the usual, spherically symmetric form,

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2(d\Omega_{N-2})^2. \quad (6)$$

by the coordinate transformation

$$dv = A(r)^{-1}(dt + \epsilon \frac{dr}{f(r)}) \quad (7)$$

In case of spherical symmetry, even when  $f(r)$  is replaced by  $f(t, r)$ , one can cast the metric in the form (5) [27].

The non-vanishing components of the Einstein tensor [24] are

$$G_r^v = \frac{(N-2)}{rA^2} \frac{\partial A}{\partial r}, \quad (8a)$$

$$G_v^r = -\frac{(N-2)}{2r} \frac{\partial f}{\partial v}, \quad (8b)$$

$$G_v^v = \frac{(N-2)}{2r^2} \left[ r \frac{\partial f}{\partial r} - (N-3)(1-f) \right], \quad (8c)$$

$$G_r^r = \frac{(N-2)}{2r^2} \left[ r \frac{\partial f}{\partial r} - (N-3)(1-f) \right] + \frac{(N-2)}{rA} f \frac{\partial A}{\partial r}, \quad (8d)$$

$$\begin{aligned} 2r^2 G_{\theta_i}^{\theta_i} = & r^2 \frac{\partial^2 f}{\partial r^2} + (N-3) \left( 2r \frac{\partial f}{\partial r} - (N-4)(1-f) \right) + 2(N-3) \frac{rf}{A} \frac{\partial A}{\partial r} + 2 \frac{r^2}{A^2} \frac{\partial^2 A}{\partial v \partial r} \\ & + 3 \frac{r^2}{A} \frac{\partial f}{\partial r} \frac{\partial A}{\partial r} + 2 \frac{r^2 f}{A} \frac{\partial^2 A}{\partial r^2} - 2 \frac{r^2}{A^3} \frac{\partial A}{\partial r} \frac{\partial A}{\partial v}, \end{aligned} \quad (8e)$$

where  $\{x^a\} = \{v, r, \theta_1, \dots, \theta_{N-2}\}$ . We shall consider the special case  $T_r^v = 0$  (hypothesis), which means from Eq. (8a),  $A(v, r) = g(v)$ . This also implies that  $G_v^v = G_r^r$  (Eq. (8d)). However, by introducing another null coordinate  $\bar{v} = \int g(v) dv$ , we can always set without the loss of generality,  $A(v, r) = 1$ . Hence, the metric takes the form,

$$ds^2 = - \left[ 1 - \frac{2m(v, r)}{(N-3)r^{(N-3)}} \right] dv^2 + 2\epsilon dv dr + r^2 (d\Omega_{N-2})^2. \quad (9)$$

Therefore the entire family of solutions we are searching for is determined by a single function  $m(v, r)$ . Henceforth, we adopt here a method similar to Salgado [17] which we modify here to accommodate the non static case. In what follows, we shall consider  $\epsilon = 1$ . The Einstein field equations are

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}, \quad (10)$$

and combining Eqs. (8) and (10), we have if  $a \neq b$ ,  $T_b^a = 0$  except for a non-zero off-diagonal components  $T_v^r$ . In addition, we observe that  $T_v^v = T_r^r$ . Thus the EMT can be written as :

$$T_b^a = \begin{pmatrix} T_v^v & T_r^v & 0 & 0 & \dots & \dots \\ T_v^r & T_r^r & 0 & 0 & \dots & \dots \\ 0 & 0 & T_{\theta_1}^{\theta_1} & 0 & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & T_{\theta_{N-2}}^{\theta_{N-2}} \end{pmatrix}.$$

which in general belongs to a Type II fluid with  $T_{\theta_1}^{\theta_1} = T_{\theta_2}^{\theta_2} = \dots = T_{\theta_{N-2}}^{\theta_{N-2}}$ . It may be recalled that EMT of a Type II fluid has a double null eigen vector, whereas an EMT of a Type I fluid has only one time-like eigen vector [28]. On the other hand, from the Einstein equations, it follows that

$$\nabla_a T_b^a = 0. \quad (11)$$

Enforcing the conservation laws  $\nabla_a T_b^a = 0$ , yields the following non-trivial differential equations:

$$\frac{\partial T_r^r}{\partial r} = -\frac{(N-2)}{r} (T_r^r - T_{\theta_1}^{\theta_1}), \quad (12)$$

and, using,  $T_r^r = T_v^v$ ,

$$\frac{\partial T_v^v}{\partial v} = -\frac{\partial T_v^r}{\partial r} - \frac{(N-2)}{r} T_v^r. \quad (13)$$

Using the assumption made above that  $T_{\theta_1}^{\theta_1} = kT_r^r$ , we obtain the following linear differential equation

$$\frac{\partial T_r^r}{\partial r} = -\frac{(N-2)}{r}(1-k)T_r^r, \quad (14)$$

which can be easily integrated to give

$$T_r^r = \frac{C(v)}{r^{(N-2)(1-k)}}, \quad (15)$$

where  $C(v)$  is an arbitrary function of  $v$ , arising as an integration constant. Then, using hypothesis (iii), we conclude that

$$T_b^a = \frac{C(v)}{r^{(N-2)(1-k)}} \text{diag}[1, 1, k, \dots, k]. \quad (16)$$

Now using Eqs. (8) [with  $f(v, r) = 1 - 2m(v, r)/(N-3)r^{(N-3)}$ ], (10) and (15), we get

$$\frac{\partial m}{\partial r} = -8\pi \left( \frac{N-3}{N-2} \right) \frac{C(v)}{r^{-(N-2)k}}, \quad (17)$$

which trivially integrates to

$$m(v, r) = \begin{cases} M(v) & \text{if } C(v) = 0, \\ M(v) - 8\pi C(v) \left( \frac{N-3}{N-2} \right) \frac{1}{(N-2)k+1} r^{(N-2)k+1} & \text{if } C(v) \neq 0 \text{ and } k \neq -1/(N-2), \\ M(v) - 8\pi C(v) \left( \frac{N-3}{N-2} \right) \ln r & \text{if } C(v) \neq 0 \text{ and } k = -1/(N-2). \end{cases} \quad (18)$$

Here the function  $M(v)$  arises as a result of integration. What remains to be calculated is the only non-zero off-diagonal component  $T_v^r$  of the EMT. From Eqs. (8) and (10), one gets

$$T_v^r = \frac{1}{8\pi r^{N-2}} \left( \frac{N-2}{N-3} \right) \frac{\partial m}{\partial v}, \quad (19)$$

which, on using Eq. (18), gives

$$T_v^r = \begin{cases} \frac{1}{8\pi r^{N-2}} \left( \frac{N-2}{N-3} \right) \frac{\partial M}{\partial v} - \frac{1}{(N-2)k+1} \frac{\partial C}{\partial v} r^{(N-2)(k-1)+1} & \text{if } k \neq -1/(N-2), \\ \frac{1}{8\pi r^{N-2}} \left( \frac{N-2}{N-3} \right) \frac{\partial M}{\partial v} - \frac{1}{r^{N-2}} \frac{\partial C}{\partial v} \ln r & \text{if } k = -1/(N-2). \end{cases} \quad (20)$$

It is seen that Eq. (13) is identically satisfied. Hence the theorem is proved. The theorem proved above represents a general class of non-static,  $N$ -dimensional spherically symmetric solutions to Einstein's equations describing radiating black-holes with the EMT which satisfies the conditions in accordance with hypothesis (iii). The solutions generated here highly rely on the assumption (iii). On the other hand, although hypothesis (iv) is not used a priori for proving the result, but it is indeed suggested by regularity of the solution at the origin, from which,  $T_v^v = T_r^r|_{r=0}$  (see [17] for further details).

The family of the  $N$ -dimensional solutions outlined here contains  $N$ -dimensional version of, for instance, Vaidya [23, 29] Bonnor-Vaidya [24, 30], dS/AdS [24], global monopole [24, 25, 31], Husain [12, 24, 25], and Harko-Cheng SQM solution [14, 23]. Obviously, by proper choice of the functions  $M(v)$  and  $C(v)$ , and  $k$ -index, one can generate as many solutions as required. The above solutions include most of the known Vaidya-based spherically symmetric solutions of the Einstein field equations. When  $N = 4$ , the 4D solutions derived in [16, 18] can be recovered. The static black holes solutions, in both HD [32] and in 4D [17], can be recovered by setting  $M(v) = M$ ,  $C(v) = C$ , with  $M$  and  $C$  as constants, in which case matter is Type I.

In summary, we have shown that the metric

$$ds^2 = - \left[ 1 - \frac{2M(v)}{(N-3)r^{N-3}} + \frac{16\pi C(v)}{(N-2)[(N-2)k+1]} r^{(N-2)(k-1)+2} \right] dv^2 + 2dvdr + r^2(d\Omega_{N-2})^2, \quad (21)$$

is a solution of the Einstein equations for the stress energy tensor Eqs. (3) and (4). A metric is considered to be asymptotically flat if in the vicinity of a spacelike hypersurface its components behave as

$$g_{ab} \rightarrow \eta_{ab} + \frac{\alpha_{ab}(x^c/r, t)}{r} + \mathcal{O}\left(\frac{1}{r^{1+\epsilon}}\right), \quad (22)$$

as  $r \rightarrow \infty$ . ( $\epsilon > 0$ ,  $\eta_{ab}$  is the Minkowski metric,  $\alpha_{ab}$  is an arbitrary symmetric tensor, and  $x^c$  is a flat coordinate system at spacelike infinity). According to this definition, our metrics Eq. (1) are asymptotically flat for  $k > -1/(N-2)$  and are cosmological for  $k < -1/(N-2)$ . In particular, for  $k=-1$ ,  $M(v) = M$  and  $2C(v) = Q^2$ , the metric is just higher dimensional Reissner-Nordström. The detailed of the asymptotic structure of spatial infinity in higher-dimensional space-times can be found in Ref. [33] and the different conformal diagrams for maximal extension of 4D Vaidya is discussed by Fayos *et al.* [34].

The theorem proved shows that rather than a mathematical coincidence, the above form of the metric is a consequence of the features of the energy-momentum tensor considered. In the above exact solutions, the associated energy momentum tensors share some properties that are taken into account in the theorem in a general fashion without specifying the nature of the matter. Therefore, the theorem helps to characterize a whole two-parameter family of solutions to the Einstein field equations.

The solutions discussed in the section are characterized by two arbitrary functions  $M(v)$  and  $C(v)$ , and the cosmological constant  $\Lambda$ . Thus one would like to generalize the above theorem to include  $\Lambda$ . We can show that the energy momentum tensor components, in general, can be written as,  $T_b^a = T_{(f)b}^a - \frac{\Lambda}{8\pi}\delta_b^a$  [17, 22], where  $\Lambda$  is the cosmological constant and  $T_{(f)b}^a$  is energy momentum tensor of the matter fields that satisfy  $T_{(f)\theta}^\theta = kT_{(f)r}^r$ . A trivial extension of the theorem allows one to cover a three-parameter family of solutions, with one of the parameters being a cosmological constant  $\Lambda$ . Next, we just state (proof being similar) the generalization of the Theorem I.

**Theorem - II:** Let  $(M, g_{ab})$  be an  $N$ -dimensional space-time [ $\text{sign}(g_{ab}) = +(N-2)$ ] such that (i) It is non-static and spherically symmetric, (ii) it satisfies Einstein field equations, (iii) the total energy-momentum tensor is given by  $T_b^a = T_{(f)b}^a - \frac{\Lambda}{8\pi}\delta_b^a$ , where  $\Lambda$  is the cosmological constant and  $T_{(f)b}^a$  is energy momentum tensor of the matter fields, (iv) in the Eddington coordinates where  $ds^2 = -A(v, r)^2 dv^2 + 2A(v, r) dv dr + r^2(d\Omega_{N-2})^2$ , the EMT  $T_{(f)b}^a$  satisfies the conditions  $T_{(f)r}^v = 0$ ,  $T_{(f)\theta_1}^{\theta_1} = kT_{(f)r}^r$ , ( $k = \text{const.} \in \mathcal{R}$ ), (v) it possesses a regular Killing horizon or a regular origin. Then the metric of the space-time is given by metric (1), where

$$m(v, r) = \begin{cases} M(v) + \frac{(N-3)}{(N-2)(N-1)}\Lambda r^{N-1} & \text{if } C(v) = 0, \\ M(v) - 8\pi C(v) \left(\frac{N-3}{N-2}\right) \frac{1}{(N-2)k+1} r^{(N-2)k+1} + \frac{(N-3)}{(N-2)(N-1)}\Lambda r^{N-1} & \text{if } C(v) \neq 0 \text{ and } k \neq -1/(N-2), \\ M(v) - 8\pi C(v) \left(\frac{N-3}{N-2}\right) \ln r + \frac{(N-3)}{(N-2)(N-1)}\Lambda r^{N-1} & \text{if } C(v) \neq 0 \text{ and } k = -1/(N-2). \end{cases} \quad (23)$$

$$T_b^a = \frac{C(v)}{r^{(N-2)(1-k)}} \text{diag}[1, 1, k, \dots, k] - \frac{\Lambda}{8\pi} \text{diag}[1, 1, 1, \dots, 1] \quad (24)$$

and

$$T_v^r = \begin{cases} \frac{1}{8\pi r^{N-2}} \left(\frac{N-2}{N-3}\right) \frac{\partial M}{\partial v} - \frac{1}{(N-2)k+1} \frac{\partial C}{\partial v} r^{(N-2)(k-1)+1} & \text{if } k \neq -1/(N-2), \\ \frac{1}{8\pi r^{N-2}} \left(\frac{N-2}{N-3}\right) \frac{\partial M}{\partial v} - \frac{1}{r^{N-2}} \frac{\partial C}{\partial v} \ln r & \text{if } k = -1/(N-2). \end{cases} \quad (25)$$

Here,  $M(v)$  and  $C(v)$  are arbitrary functions of  $v$ , arising as integration constants, whose values depend on the boundary conditions and the fundamental constants of the underlying matter.

### III. ENERGY CONDITIONS

The family of solutions discussed here, in general, belongs to Type II fluid defined in [28]. When  $m = m(r)$ , we have  $\mu=0$ , and the matter field degenerates to type I fluid [16]. In the rest frame associated with the observer, the

energy-density of the matter will be given by (assuming  $\Lambda = 0$ ),

$$\mu = T_v^r, \quad \rho = -T_t^t = -T_r^r = -\frac{C(v)}{r^{(N-2)(1-k)}}, \quad (26)$$

and the principal pressures are  $P_i = T_i^i$  (no sum convention). Therefore  $P_r = T_r^r = -\rho$  and  $P_{\theta_1} = kP_r = -k\rho$  (hypothesis (iii)).

a) *The weak energy conditions* (WEC): The energy momentum tensor obeys inequality  $T_{ab}w^aw^b \geq 0$  for any timelike vector, i.e.,

$$\mu \geq 0, \quad \rho \geq 0, \quad P_{\theta_1} = P_{\theta_2} = \dots = P_{\theta_{(N-2)}} \geq 0. \quad (27)$$

We say that strong energy condition (SEC), holds for Type II fluid if, Eq. (27) is true., i.e., both WEC and SEC, for a Type II fluid, are identical.

b) *The dominant energy conditions* : For any timelike vector  $w_a$ ,  $T^{ab}w_aw_b \geq 0$ , and  $T^{ab}w_a$  is non-spacelike vector, i.e.,

$$\mu \geq 0, \quad \rho \geq P_{\theta_1}, P_{\theta_2} = \dots = P_{\theta_{(N-2)}} \geq 0. \quad (28)$$

Clearly, (a) is satisfied if  $C(v) \leq 0, k \leq 0$ . However,  $\mu > 0$  gives the restriction on the choice of the functions  $M(v)$  and  $C(v)$ . From Eq. (4), ( $k \neq -1/(n-2)$ ), we observe  $\mu > 0$  requires,

$$\frac{1}{8\pi r^{N-2}} \left( \frac{N-2}{N-3} \right) \frac{\partial M}{\partial v} - \frac{1}{(N-2)k+1} \frac{\partial C}{\partial v} r^{(N-2)(k-1)+1} > 0. \quad (29)$$

This, in general, is satisfied, if

$$\left( \frac{N-2}{N-3} \right) \frac{\partial M}{\partial v} > 0, \text{ and, either } \frac{\partial C}{\partial v} > 0 \text{ and } k < -1/(N-2), \text{ or } \frac{\partial C}{\partial v} < 0 \text{ and } k > -1/(N-2). \quad (30)$$

On the other hand, for  $k = -1/(N-2)$ ,  $\mu \geq 0$  if  $\partial M / \partial C \geq 8\pi \left( \frac{N-3}{N-2} \right) \ln r$ . The DEC holds if  $C(v) \leq 0$  and  $-1 \leq k \leq 0$ , and the function  $M$  is subject to the condition (30). Clearly,  $0 \leq -k \leq 1$ .

#### IV. SINGULARITY AND HORIZONS

The invariants are regular everywhere except at the origin  $r = 0$ , where they diverge. Hence, the space-time has the scalar polynomial singularity [28] at  $r = 0$ . The nature (a naked singularity or a black hole) of the singularity can be characterized by the existence of radial null geodesics emerging from the singularity. The singularity is at least locally naked if there exist such geodesics, and if no such geodesics exist, it is a black hole. The study of causal structure of the space-time is beyond the scope of this paper and will be discussed elsewhere.

In order to further discuss the physical nature of our solutions, we introduce their kinematical parameters. Following York [35] a null-vector decomposition of the metric (1) is made of the form

$$g_{ab} = -n_al_b - l_an_b + \gamma_{ab}, \quad (31)$$

where,

$$n_a = \delta_a^v, \quad l_a = \frac{1}{2} \left[ 1 - \frac{2m(v, r)}{(N-3)r^{N-3}} \right] \delta_a^v + \delta_a^r, \quad (32a)$$

$$\gamma_{ab} = r^2 \delta_a^{\theta_1} \delta_b^{\theta_1} + r^2 \left[ \left( \prod_{j=1}^{i-1} \sin^2(\theta_j) \right) \right] \delta_a^{\theta_i} \delta_b^{\theta_i}, \quad (32b)$$

$$l_al^a = n_an^a = 0, \quad l_an^a = -1, \\ l^a \gamma_{ab} = 0; \gamma_{ab} n^b = 0, \quad (32c)$$

with  $m(v, r)$  given by Eq. (2). The optical behavior of null geodesics congruences is governed by the Carter [36] form of the Raychaudhuri equation

$$\frac{d\Theta}{dv} = \mathcal{K}\Theta - R_{ab}l^a l^b - (\gamma_c^c)^{-1}\Theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab}, \quad (33)$$

with expansion  $\Theta$ , twist  $\omega$ , shear  $\sigma$ , and surface gravity  $\mathcal{K}$ . Here  $R_{ab}$  is the  $N$ -dimensional Ricci tensor,  $\gamma_c^c$  is the trace of the projection tensor for null geodesics. The expansion of the null rays parameterized by  $v$  is given by

$$\Theta = \nabla_a l^a - \mathcal{K}, \quad (34)$$

where the  $\nabla$  is the covariant derivative. In the present case,  $\sigma = \omega = 0$  [35], and the surface gravity is,

$$\mathcal{K} = -n^a l^b \nabla_b l_a. \quad (35)$$

Spherically symmetric irrotational space-times, such as under consideration, are vorticity and shear free. The structure and dynamics of the horizons are then only dependent on the expansion,  $\Theta$ . As demonstrated by York [35], horizons can be obtained by noting that (i) apparent horizons are defined as surface such that  $\Theta \simeq 0$  and (ii) event horizons are surfaces such that  $d\Theta/dv \simeq 0$ . Substituting Eqs. (23), (32) and (35) into Eq. (34), we get, ( $k \neq -1/2$ )

$$\Theta = \frac{1}{r} \left[ 1 - \frac{2\overline{M}(v)}{r^{N-3}} + Q^2(v)r^{(N-2)k-(N-4)} - \chi^2 r^2 \right], \quad (36)$$

where,

$$\overline{M}(v) = \frac{M(v)}{(N-3)}, \quad \chi^2 = \frac{2\Lambda}{(N-2)(N-1)}, \quad Q^2(v) = \frac{16\pi C(v)}{(N-2)((N-2)k+1)}. \quad (37)$$

Since the York conditions require that at apparent horizons  $\Theta$  vanish, it follows from the Eq. (36) that apparent horizons will satisfy

$$\chi^2 r^{N-1} - Q^2 r^{(N-2)k+1} - r^{N-3} + 2\overline{M}(v) = 0, \quad (38)$$

which in general has two positive solutions. For  $\chi^2 = Q^2 = 0$ , we have Schwarzschild horizon  $r = (2\overline{M})^{\frac{1}{N-3}}$ , and for  $M = Q^2 = 0$  we have de Sitter horizon  $r = 1/\chi$ . As mentioned above, for  $k = -1$ , one gets Bonnor-Vaidya solution, in which case the various horizons are identified and analyzed by Mallett [37] and hence, to conserve space, we shall avoid the repetition of same. For general  $k$ , as it stands, Eq. (38) will not admit simple closed form solutions. However, for

$$Q^2 = Q_c^2 = \frac{-(N-3)}{((N-2)k+1)} \left[ \frac{2M((N-2)k+1)}{(N-3)((N-2)k-(N-4))} \right]^{\frac{(N-4)-(N-2)k}{(N-3)}}, \quad (39)$$

with  $\chi^2 = 0$ , the two roots of the Eq. (38) coincide and there is only one horizon

$$r = \left[ \frac{2M((N-2)k+1)}{(N-3)((N-2)k-(N-4))} \right]^{\frac{1}{(N-3)}} \quad (40)$$

For  $Q^2 \leq Q_c^2$  there are two horizons, namely a cosmological horizon and a black hole horizon. On the other hand if, the inequality is reversed,  $Q^2 > Q_c^2$  no horizon would form.

## V. CONCLUDING REMARKS

In the study of the Einstein equations in the 4D space-time several powerful mathematical tools were developed, based on the space-time symmetry, algebraical structure of space-time, internal symmetry and solution generation technique, global analysis, and so on. It would be interesting how to develop some of these methods to higher dimensional space-time. With this as motivation, plus the fact that exact solutions are always desirable and valuable, we have extended to higher dimensional space-time, a recent theorem [18] and its trivial extension (that includes cosmological term  $\Lambda$ ), which, with certain restrictions on the EMT, characterizes a large family of radiating black hole

solutions in  $N$ -dimensions, representing, in general, spherically symmetric Type II fluid. In particular, the monopole-de Sitter-charged Vaidya and Husain solutions can be generated from our analysis and when  $n = 4$ , one recovers the 4D black solutions. If  $M = C = \text{constant}$ , we have  $\mu=0$ , and the matter field degenerates to type I fluid and we can generate static black hole solutions by a proper choice of these constants. Since many known solutions are identified as particular case of this family and hence it would be interesting to ask whether there exist realistic matter that follows the restrictions of the theorem that would generate a new black hole solution.

The solutions depend on one parameter  $k$ , and two arbitrary functions  $M(v)$  and  $C(v)$  (modulo energy conditions). It is possible to generate various solutions by proper choice of these functions and parameter  $k$ . Further, most of the known static spherically symmetric black hole solutions, in 4D and HD, can be recovered from our analysis.

It should be interesting to apply these metrics to study the gravitation collapse and naked singularities formation. Finally, the result obtained would also be relevant in the context of string theory which is often said to be next "theory of everything" and in the study of gravitational collapse.

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